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1992 J. Phys. A: Math. Gen. 25 4941

(http://iopscience.iop.org/0305-4470/25/18/024)

J. Phys. A: Math. Gen. **25** (1992) 4941–4946. Printed in the UK

Dynamical breaking of supersymmetry by fermion pair condensation*

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Received 5 November 1991, in final form 22 April 1992

Abstract. Supersymmetry breaking is studied without adding the parity-violating term. It is shown, in terms of the Nambu-Jona-Lasinio mechanism, that supersymmetry breaking can be realized by the fermion pair condensation $\langle\bar{\psi}(x)\psi(x)\rangle$ even in the simplest supersymmetric theory, and there is no Goldstone fermion in our model.

It is well known that spontaneous symmetry breaking plays an important role in quantum field theories and in solid state physics applied to such phenomena as ferromagnetism, superfluidity and superconductivity. A symmetry transformation of a theory is described as spontaneously broken when the ground state of the system is not invariant under this transformation.

Another important type of symmetry breaking is so-called dynamical symmetry breaking [1]. Dynamical breaking of a symmetry is realized by a mechanism which is not explicitly present in the original Lagrangian, of which the Nambu–Jona-Lasinio mechanism [1] is an example. Nambu and Jona-Lasinio have suggested that the nucleon mass arises largely as a self-energy of some primary massless fermions. Nambu–Jona-Lasinio theory is based on the Lagrangian density

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_\mathrm{I} \tag{1}$$

where $\mathscr{L}_0$ is a free Lagrangian,

$$\mathscr{L}_0=\bar{\psi}\mathrm{i}\partial\!\!\!/\psi \tag{2}$$

and $\mathscr{L}_\mathrm{I}$ is a four-fermion interaction of the type

$$\mathscr{L}_\mathrm{I}=\tfrac{1}{2}g[(\bar{\psi}\psi)^2-(\bar{\psi}\gamma_5\psi)^2]. \tag{3}$$

The crucial point of the Nambu–Jona-Lasinio mechanism is that instead of diagonal $\mathscr{L}_0$ and treating $\mathscr{L}_\mathrm{I}$ as a perturbation, they have introduced a self-energy Lagrangian,

$$\mathscr{L}_\mathrm{S}=\delta m\bar{\psi}\psi \tag{4}$$

and rewritten the Lagrangian (1) as

$$\mathscr{L}=\mathscr{L}'_0+\mathscr{L}'_\mathrm{I} \tag{5}$$

* Project supported by the National Nature Science Foundation of China.

with

$$\mathscr{L}_0' = \mathscr{L}_0 - m\bar{\psi}\psi = \bar{\psi}(\mathrm{i}\not{\partial} - m)\psi \tag{6}$$

$$\mathscr{L}_\mathrm{I}' = \mathscr{L}_\mathrm{I} + \delta m\bar{\psi}\psi = \tfrac{1}{2}g[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] + \delta m\bar{\psi}\psi. \tag{7}$$

The self-mass δm is given by

$$\delta m = \textstyle\sum^* (p)|_{\not{p}=m} \tag{8}$$

and the right-hand side of equation (8) can be calculated, to the first order in g, from the self-energy diagram of the fermion. Since $m_0 = 0$, we have $m = \delta m$ and so one obtains the self-consistent equation for the physical mass of the fermion:

$$m = \textstyle\sum^* (p)|_{\not{p}=m}. \tag{9}$$

Nambu and Jona-Lasinio evaluated

$$m \simeq 2\mathrm{i}g \operatorname{Tr} S_\mathrm{F}(0) \tag{10}$$

where

$$S_\mathrm{F}(0) = -\mathrm{i}\langle\psi(x)\bar{\psi}(x)\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{\not{p} - m}$$

represents the contribution of the closed-loop (self-energy) diagram. Equation (10) gives

$$\frac{2\pi^2}{g\Lambda^2} = 1 - \frac{m^2}{\Lambda^2} \ln\left(\frac{\Lambda^2}{m^2} + 1\right) \tag{11}$$

by introducing the invariant momentum cut-off $p^2 = \Lambda^2$.

We see that in the Nambu–Jona-Lasinio theory starting from a massless fermion one generates the physical mass of the fermion self-consistently.

Lurie and Macfarlane have shown [2] the equivalence between the Lagrangian field theory of the four-fermion type considered by Nambu and Jona-Lasinio and a Lagrangian theory of the same fermion fields with Yukawa-type coupling,

$$\mathscr{L} = \mathscr{L}_0 + G\bar{\psi}\psi\varphi_s + G\bar{\psi}\gamma_5\psi\varphi_p \tag{12}$$

and obtained the fermion mass in the equivalent Yukawa theory in the same way.

Many authors [3–5] have discussed dynamical breaking of chiral symmetry in terms of the Nambu–Jona-Lasinio mechanism. Most recently, the Nambu–Jona-Lasino mechanism has become popular as a simplified model to investigate various aspects of low-energy hadron dynamics [5].

The purpose of the present paper is to discuss supersymmetry breaking by using the Nambu–Jona-Lasinio mechanism and to show that supersymmetry breaking can be realized dynamically without adding the Fayet–Iliopoulos term to the Lagrangian. The spontaneous breaking of supersymmetry has been studied by many authors [6–10]. The usual method of spontaneous supersymmetry breaking is by adding a parity-violating term ξD (the Fayet–Iliopoulos term) to the Lagrangian. The adding of the Fayet–Iliopoulos term leads to mass splitting between bosons and fermions in the supermultiplet and hence induces supersymmetry breaking. We will show in terms of the Nambu–Jona-Lasinio mechanism mentioned above that the contribution of the self-consistency equation (see equation (41)) leads to a non-vanishing vacuum expectation value of the scalar field and provides different dynamical masses to the bosons and fermions in the supermultiplet, and the supersymmetry breaking can be realized dynamically even in the simplest supersymmetric theory.

Consider now the simplest two-dimensional supersymmetric theory of a single real supermultiplet (A, ψ, F) with the Lagrangian [11]

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu A)^2+\tfrac{1}{2}\mathrm{i}\bar{\psi}\not{\partial}\psi+\tfrac{1}{2}F^2-gFA^2-gA\bar{\psi}\psi \tag{13}$$

where $A(x)$ is a real scalar, $\psi(x)$ is a real spinor and $F(x)$ is the auxiliary field. The Lagrangian (13) is invariant under the supertransformations

$$\delta A=\bar{\varepsilon}\psi \tag{14}$$

$$\delta\psi=(-\mathrm{i}\not{\partial}A-F)\varepsilon \tag{15}$$

$$\delta F=\mathrm{i}\bar{\varepsilon}\not{\partial}\psi \tag{16}$$

where ε is a constant Grassmann parameter.

From the Lagrangian (13) we obtain the equations of motion

$$(\Box+2gF)A=-g\bar{\psi}\psi \tag{17}$$

$$F=gA^2 \tag{18}$$

$$(\tfrac{1}{2}\mathrm{i}\not{\partial}-gA)\psi=0. \tag{19}$$

By using the equations of motion we can eliminate the auxiliary field F from the Lagrangian (13), and the Lagrangian (13) can be written as

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu A)^2+\tfrac{1}{2}\mathrm{i}\bar{\psi}\not{\partial}\psi-\tfrac{1}{2}g^2A^4-gA\bar{\psi}\psi. \tag{20}$$

Let us now introduce the external sources $J_A(x)$, $J_\psi(x)$ and $J_{\bar{\psi}}(x)$ coupled to the fields $A(x)$, $\psi(x)$ and $\bar{\psi}(x)$, respectively. Then the Lagrangian (20) becomes

$$\mathscr{L}[J]=\tfrac{1}{2}(\partial_\mu A)^2+\tfrac{1}{2}\mathrm{i}\bar{\psi}\not{\partial}\psi-\tfrac{1}{2}g^2A^4-gA\bar{\psi}\psi+J_AA+\bar{J}_\psi\psi+\bar{\psi}J_{\bar{\psi}} \tag{21}$$

and the functional integral is given by

$$W[J]=\int[\mathrm{d}A][\mathrm{d}\bar{\psi}][\mathrm{d}\psi]\exp\left(\frac{\mathrm{i}}{\hbar}\int\mathrm{d}^2x\,\mathscr{L}[J]\right). \tag{22}$$

The equations of motion turn out to be

$$\Box A+2g^2A^3+g\bar{\psi}\psi=J_A(x) \tag{23}$$

$$(\tfrac{1}{2}\mathrm{i}\not{\partial}-gA)\psi=-J_\psi(x). \tag{24}$$

In general case, the vacuum expectation values

$$\langle A\rangle_0^J=\frac{\langle 0\text{ out}|A|0\text{ in}\rangle^J}{\langle 0\text{ out}|0\text{ in}\rangle^J}=\frac{\delta Z[J]}{\delta J_A(x)} \tag{25}$$

where

$$Z[J]=\frac{\hbar}{\mathrm{i}}\ln W[J] \tag{26}$$

$$J(x)=(J_A(x),J_\psi(x),\bar{J}_{\bar{\psi}}(x)) \tag{27}$$

are the functions of spacetime x. In order to derive the self-consistency condition [12-13] for field A, we now restrict ourselves to constant sources J_A, J_ψ, and $J_{\bar{\psi}}$; then the translation invariance is restored and $\langle A\rangle_0^J$ is spacetime constant, as in the source-free case.

Taking the vacuum expectation value of equation (23), one gets

$$\Box\langle A\rangle_0^J+2g^2\langle A^3\rangle_0^J+g\langle\bar\psi\psi\rangle_0^J=J_A. \tag{28}$$

The second term of equation (28) can be written as

$$\langle A^3\rangle_0^J=(\langle A\rangle_0^J)^3+3\frac{\hbar}{\mathrm{i}}\langle A\rangle_0^J\frac{\delta\langle A\rangle_0^J}{\delta J_A}+\left(\frac{\hbar}{\mathrm{i}}\right)^2\frac{\delta^2\langle A\rangle_0^J}{\delta J_A^2}. \tag{29}$$

From equation (29) we see that for the lowest order of $\hbar$, $\langle A^3\rangle_0^J$ can be approximated by $(\langle A\rangle_0^J)^3$ and so equation (28) leads to

$$(\langle A\rangle_0)^3=-\frac{1}{2g}\langle\bar\psi\psi\rangle_0=\frac{\mathrm{i}}{2g}\,\mathrm{Tr}\,S_\mathrm{F}(0) \tag{30}$$

in the limit $J\to 0$, where

$$S_\mathrm{F}(0)=-\mathrm{i}\langle\psi(x)\bar\psi(x)\rangle \tag{31}$$

is the fermion propagator. The right-hand side of equation (30) corresponds to the contribution of the closed-loop diagram (the tadpole diagram) of the fermion.

On the other hand, in equation (24), if we write the T-product as

$$T\psi(x)\bar\psi(y)=\tfrac{1}{2}[\psi(x),\bar\psi(y)]+\tfrac{1}{2}\varepsilon(x_0-y_0)\{\psi(x),\bar\psi(t)\} \tag{3‘}$$

then we obtain the equation for the propagator of the fermion, $S_\mathrm{F}(x,y)^J$, in the presence of a spacetime-dependent external source:

$$\begin{aligned}\mathrm{i}\not\partial_x S_\mathrm{F}(x,y)^J&=\delta^2(x-y)-2g\mathrm{i}\langle TA(x)\psi(x)\bar\psi(y)\rangle_0^J+2J_\psi\mathrm{i}\langle\bar\psi(y)\rangle_0^J\\&=\delta^2(x-y)+2g\left(S_\mathrm{F}(x,y)^J\langle A(x)\rangle_0^J+\frac{\hbar}{\mathrm{i}}\frac{\delta S_\mathrm{F}(x,y)^J}{\delta J_A(x)}+2\mathrm{i}J_\psi\langle\bar\psi(y)\rangle_0^J\right)\end{aligned} \tag{33}$$

where we have used [12]

$$-\mathrm{i}\langle TA(x')\psi(x)\bar\psi(y)\rangle_0^J=S_\mathrm{F}(x,y)^J\langle A(x')\rangle_0^J+\frac{\hbar}{\mathrm{i}}\frac{\delta S_\mathrm{F}(x,y)^J}{\delta J_A(x')}. \tag{34}$$

Thus, one finally obtains the Schwinger equation for $S_\mathrm{F}(x,y)^J$:

$$(\mathrm{i}\not\partial_x-2g\langle A(x)\rangle_0^J)S_\mathrm{F}(x,y)^J=\delta^2(x-y)-2J_\psi\langle\bar\psi(y)\rangle_0^J$$

or, in the limit $J\to 0$,

$$(\mathrm{i}\not\partial_x-2gA_0)S_\mathrm{F}(x-y)=\delta^2(x-y) \tag{35}$$

with

$$A_0\equiv\langle A(x)\rangle_0. \tag{36}$$

So, in momentum space, the fermion propagator can be written as

$$S_\mathrm{F}(p)=\frac{1}{\not P-2gA_0} \tag{37}$$

and we obtain from equation (30) that

$$A_0^3=\frac{\mathrm{i}}{2g}\,\mathrm{Tr}\,S_\mathrm{F}(0)=\frac{\mathrm{i}}{2g}\,\mathrm{Tr}\int\frac{\mathrm{d}^2P}{(2\pi)^2}\frac{1}{\not P-2gA_0}=2\mathrm{i}A_0\int\frac{\mathrm{d}^2P}{(2\pi)^2}\frac{1}{P^2-4g^2A_0^2} \tag{38}$$

or

$$m_\psi^2 = 8\mathrm{i}g^2 \int \frac{\mathrm{d}^2P}{(2\pi)} \frac{1}{P^2 - m_\psi^2} \tag{39}$$

with

$$m_\phi = 2gA_0 \tag{40}$$

which is the mass of the fermion field.

Equation (39) is divergent, but with a momentum cut-off Λ it can be made finite. From equation (38) we see that the vacuum expectation value of the scalar field A has two solutions: either $A_0 = 0$, or

$$A_0^2 = 2\mathrm{i} \int \frac{\mathrm{d}^2P}{(2\pi)^2} \frac{1}{P^2 - 4g^2A_0^2}. \tag{41}$$

The first, trivial, solution corresponds to the ordinary perturbation result. The second, non-trivial, solution implies supersymmetry breaking, because the non-vanishing value of A_0 will induce different masses for the boson with fermion in the model. Equation (39) will determine the fermion mass in terms of the coupling constant g and the momentum cut-off Λ.

Translating the scalar field A as

$$A \to A' + A_0 \tag{42}$$

then the Lagrangian (20) becomes

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu A')^2 - \tfrac{1}{2}m_A^2 A'^2 + \tfrac{1}{2}\mathrm{i}\bar{\psi}\not{\partial}\psi - \tfrac{1}{2}m_\psi\bar{\psi}\psi - 2g^2A_0^2A'^3 - \tfrac{1}{2}g^2A'^4 - gA'\bar{\psi}\psi - 2g^2A_0^3A' \tag{43}$$

where

$$m_A^2 = 6A_0^2g^2 \tag{44}$$

$$m_\psi = 2gA_0 \tag{45}$$

are the masses of the scalar field A' and the fermion field ψ, respectively. We see that in terms of the Nambu-Jona-Lasinio mechanism the fermion and boson have acquired different masses dynamically and so the supersymmetry is broken. Note that the vacuum expectation value of the scalar field is determined by the fermion pair condensation $\langle\bar{\psi}\psi\rangle$, which gives a measure of the degree of breaking of the symmetry [4]. So we conclude that the supersymmetry is broken dynamically by the fermion pair condensation.

It should be noted that in the lattice QCD, the pion mass also turn out to be proportional to the fermion pair condensation. One of the authors [14] has constructed the coupled equation of the Green function for composite fields in lattice QCD and has shown that the pion mass is determined not only by the light quark mass but also by the fermion pair condensation $\langle\bar{\psi}\psi\rangle$, that is, the pion obtains its own mass via the fermion pair condensation $\langle\bar{\psi}\psi\rangle$. This fact, together with our result in this paper, implies that besides the Higgs mechanism the dynamical breaking of a symmetry by the fermion pair condensation is one of the ways providing mass for a field.

From equation (18) we obtain

$$\langle F\rangle_0 = g\langle A^2\rangle_0 \neq 0 \tag{46}$$

namely the vacuum expectation value of the auxiliary field is non-zero. This is further evidence for supersymmetry breaking.

Note that, in our model, there is no Goldstone fermion for the supersymmetry breakdown. This result is very similar to symmetry breaking the linear σ-model [15]. In the linear σ-model the linear term $c\sigma$ induces symmetry breaking (externally) but there is no Goldstone boson. The term $gA\bar{\psi}\psi$ in our model is analoguous to the linear term $c\sigma$ in the σ-model and $g\langle\bar{\psi}\psi\rangle$ corresponds to the coefficient c in the σ-model.

Girardello *et al* [16] have also claimed that there are no Goldstone fermions associated with supersymmetry breaking.

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